

Black Holes in Higher Dimensions (Black Strings and Black Rings)

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The last three years have again seen new exciting developments in the area of higher dimensional black objects. For black objects with noncompact higher dimensions, the solution space was explored further within the blackfold approach and with numerical schemes, yielding a large variety of new families of solutions, while limiting procedures created so-called super-entropic black holes. Concerning compact extra dimensions, the sequences of static nonuniform black strings in five and six dimensions were extended to impressively large values of the nonuniformity parameter with extreme numerical precision, showing that an oscillating pattern arises for the mass, the area or the temperature, while approaching the conjectured double-cone merger solution. Besides the presentation of interesting new types of higherdimensional solutions, also their physical properties were addressed in this session. While the main focus was on Einstein gravity, a significant number of talks also covered Lovelock theories.

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1. Introduction

Starting with Kaluza and Klein the presence of higher dimensions has been a recurring theme in our quest for a unified theory of the fundamental interactions, culminating in the formulation of string theory with its additional dimensions needed for its consistency. With respect to gravity alone one often considers the number of spacetime dimensions D as a parameter of the theory, where the study of the dependence on this parameter may lead to new insights into the theory. Concerning black holes this study has indeed brought forward many new and unexpected features, which could not have been anticipated given our knowledge of black holes in four dimensions.

When a compact flat direction is added to a Schwarzschild black hole a black string emerges, where the horizon completely wraps the compact extra dimension. The horizon topology is then given by $S^2 \times S^1$, where the S^1 represents the compact dimension. When the length L of the compact dimension is large as compared to the Schwarzschild radius r_H of the black hole, the black string is thin and unstable as demonstrated by Gregory and Laflamme¹ long ago. For a given radius r_H there is a critical length L_{cr} , where the stability changes, so a fat black string is stable. At the critical point a new branch of black string solutions emerges, which are no longer uniform w.r.t. the compact coordinate^{2–5}. This branch of nonuniform black strings has now been extended⁶ far closer to the topology changing merger point^{7,8}, where a branch of caged black holes is encountered^{9–12}. By adding more compact dimensions black branes are obtained, which also suffer from the Gregory-Laflamme instability.

In the case of noncompact extra dimensions the Kerr solution was generalized by Myers and Perry¹³, obtaining rotating black holes with spherical horizon topology. Interestingly, in six and more dimensions, there is no upper bound on the angular momentum of the singly spinning Myers-Perry black holes for a given mass. This led Emparan and Myers¹⁴ to the conjecture, that there should be a dynamical bound of the angular momentum. Based on the geometry of the highly flattened horizon they argued that these overrotating black holes should develop Gregory-Laflamme type instabilities, giving rise to new branches of black holes, whose horizon would be deformed. Indeed, such pinched black holes have been found recently¹⁵ showing the consistency of the envisaged phase diagram^{16,17}.

In six dimensions this phase diagram includes black rings, which come with two branches¹⁸, a thin black ring branch and a fat black ring branch, similar to the case of five dimensions. However, here the fat black ring branch does not extend to zero area, but is expected to end at a horizon topology changing merger solution with a finite area, where it merges with a branch of pinched black holes. Also the phase diagram for a more general set of solutions has been studied recently, where the black ring is generalized to incorporate a horizon topology $S^2 \times S^3$, which has been dubbed black ringoid^{19,20}.

While those ringlike solutions were obtained numerically, their existence had been predicted within the blackfold approach^{21–23}. This approach needs two different scales which in this context are given by the large size of the ring and the small size of the black hole horizon. Comparison of the numerical solutions with the blackfold predictions showed, that the blackfold remains reliable far longer than expected, giving still acceptable predictions as the branch of fat black rings/fat ringoids is approached. It finally starts to fail in the vicinity of the cusp, where the thin and the fat black ring/black ringoid branches merge.

While the blackfold approach predicts a large number of new solutions, there may be further solutions, which do not satisfy its basic assumptions, but which might exist nevertheless as part of the phase space of higherdimensional black objects. A nice introduction to blackfolds and to some of its intriguing beasts was the first highlight of the session. Another set of intriguing black holes presented possess at the same time a noncompact horizon, which is topologically a higher-dimensional sphere with two punctures, and a finite area. Violating the isoperimetric inequality conjecture, these black holes have been named super-entropic^{24–27}.

But also seemingly boring higherdimensional black holes with spherical horizon topology were recently observed to exhibit fascinating new features²⁸. The prerequisite here was the presence of a gauge field with a Chern-Simons term. When the Chern-Simons coupling is sufficiently large, sequences of radially excited black holes arise, there is violation of uniqueness between extremal and nonextremal black holes, and near-horizon solutions are seen to correspond to an infinite sequence of global black holes, a single global black hole, or no black hole at all.

As discussed in detail in the following in section 2, the session covered many

new exciting results on black holes in Einstein gravity. Moreover, Lovelock gravity and its interesting black holes were a second major topic of the session, as discussed in section 3.

2. Solutions in higherdimensional Einstein gravity

In five spacetime dimensions there are a number of construction methods to obtain exact solutions. However, there does not seem to exist an analytic framework in order to obtain solutions in more than five dimensions. Consequently, one has to either resort to perturbative methods or to numerical methods at the moment.

In the following we will discuss the various types of new solutions of higherdimensional black objects in Einstein gravity, that were presented in the session.

2.1. *Solutions with nonspherical horizon topology*

Starting with black objects in noncompact extra dimensions we will then turn to discuss black objects with compact extra dimensions.

2.1.1. *Solutions in the blackfold approach*

In their seminal paper¹³ Myers and Perry suggested a heuristic way to construct black rings: take a black string, bend it, and spin it along the ring S^1 direction to achieve balance^{29,30}. This heuristic picture forms the basis of the perturbative technique of matched asymptotic expansions¹⁶ and the blackfold approach^{21–23}.

The blackfold approach has been suggested to obtain perturbative solutions for black objects in more than five dimensions, when there are two scales associated with the solutions, which are sufficiently far apart^{16,21–23}. An ab initio derivation of the blackfold effective theory has been given in³¹.

In his exciting talk on *New Geometries for Black Hole Horizons: a Review of the Blackfold Approach* the invited speaker Jay Armas first explained the basic principles of the blackfold approach. In particular, he explained the building of the effective theory, and he argued how the physical properties of black objects can be captured by the physics of fluid flows^{32,33}.

Subsequently, Jay Armas concentrated on the question of how to scan for horizons. Based on his results in³⁴ he provided evidence for the existence of rather involved horizon geometries and topologies, and he argued that also plane wave spacetimes allow for a very rich phase structure of higherdimensional black holes. As a key ingredient he employed results from classical minimal surface theory. He then explicitly constructed blackfold solutions consisting of planes, helicoids, catenoids and Scherk surfaces³⁴.

Next Jay Armas considered the elastic expansion^{35,36}. He pointed out that stationary fluid brane configurations are characterized by a set of elastic, hydrodynamic and spin response coefficients, and that the elastic and hydrodynamic degrees

of freedom are coupled, and he employed the second order effective action of stationary blackfolds to obtain the higher order corrected properties of thin black rings³⁷. Indeed, for black rings in seven dimensions, the agreement of the corrected perturbative results with the exact numerical results¹⁵ is most impressive. The agreement remains excellent also in the presence of an electric charge, as shown for the case of black rings carrying Maxwell charge in Einstein-Maxwell- Dilaton theory with the Kaluza-Klein coupling constant³⁸.

In the last part of his talk Jay Armas considered worldvolume effective actions for black holes by integrating out further scales³⁹. In particular, he presented novel geometries for black hole horizons in higherdimensional asymptotically flat space-time, concentrating on helicoidal black rings in $D > 5$ and helicoidal black tori in $D = 7$.

2.1.2. Black ringoids

While the blackfold approach is excellent and well justified when the lengthscales in the problem are very different, one has to resort to numerical methods, when the length scales become comparable, in order to complete the phase diagram of higherdimensional black objects.

In his invited talk on *Black Ringoids: New Higherdimensional Black Objects with Nonspherical Horizon Topology*, Eugen Radu started by recalling the well known phase diagram in five dimensions for Myers-Perry black holes and black rings, noting that both meet in a singular solution with vanishing horizon area, and discussed the blackfold limit.

Subsequently, Eugen Radu described a general framework for obtaining black holes with $S^p \times S^q$ horizon topology, concentrating on the case of $S^{D-(2k+3)} \times S^{2k+1}$ topology, where the spinning part is the S^{2k+1} part¹⁹. For an appropriately chosen value of the horizon angular velocity, the solutions become balanced. A list of such solutions for $D \leq 11$ is shown in Table 1.

	<i>spherical horizon</i>	<i>black rings</i>	<i>black ringoids</i>		
	MP/‘pinched’	$k = 0$	$k = 1$	$k = 2$	$k = 3$
$D = 5$	S^3	$\mathbf{S}^2 \times \mathbf{S}^1$			
$D = 6$	S^4	$S^3 \times S^1$			
$D = 7$	S^5	$S^4 \times S^1$	$\mathbf{S}^2 \times \mathbf{S}^3$		
$D = 8$	S^6	$S^5 \times S^1$	$S^3 \times S^3$		
$D = 9$	S^7	$S^6 \times S^1$	$S^4 \times S^3$	$\mathbf{S}^2 \times \mathbf{S}^5$	
$D = 10$	S^8	$S^7 \times S^1$	$S^5 \times S^3$	$S^3 \times S^5$	
$D = 11$	S^9	$S^8 \times S^1$	$S^6 \times S^3$	$S^4 \times S^5$	$\mathbf{S}^2 \times \mathbf{S}^7$

Table 1. Horizon topologies for spinning balanced black objects considered in¹⁹.

Eugen Radu emphasized, that the solutions considered so far had all equal angular momenta on the rotating S^{2k+1} , the reason being, that in this case the problem

simplifies to a codimension-two problem. Moreover, he explained that in order to construct such solutions numerically, it turned out to be crucial to employ an adequate coordinate system. Then the solutions could be obtained by solving the resulting set of PDEs subject to appropriate boundary conditions, where a finite difference solver and a spectral solver were used, with very good agreement between the results of the two different methods. The appropriate value of the horizon angular velocity for balanced solutions could be found by a shooting procedure.

Setting the integer $k = 0$, one obtains higherdimensional black rings, while the case $k \geq 1$ produces new nonperturbative black objects, which Eugen Radu named black ringoids. He then continued to first recall the solutions for balanced black rings in six dimensions, pointing out the very good agreement with the blackfold approach on the one hand, and the new features of the black rings in $D \geq 6$ on the other hand¹⁸. In particular, for black rings in $D \geq 6$ the fat black ring branch does not extend to vanishing area, but at a finite value a horizon topology changing transition to 'pinched' Myers-Perry black holes should occur^{14,15}. Thus there is a fundamental difference between the case $D = 5$, where the area at the merger point of black rings and black holes vanishes, and the case $D \geq 6$.

As Eugen Radu explained, the phase diagrams of black holes and black rings in five dimensions and in six dimensions represent generic situations, which are encountered also for the more general configurations, the black ringoids^{19,20}. In particular, for the $k = 1$ black ringoids in seven dimensions the pattern of the phase diagram of the black rings in five dimensions is repeated, as can be concluded from the numerical results. A theoretical argument here is based on the behavior of the respective family of Myers-Perry solutions with two equal angular momenta and the third angular momentum vanishing. The angular momentum of this family of solutions does not extend arbitrarily far for a given mass, but a maximal value is reached for a singular configuration with vanishing area. On the other hand, in more than seven dimensions the respective Myers-Perry black holes with two equal angular momenta become overrotating and instabilities should arise, associated with 'pinched' black holes at a merger solution with finite area. Therefore the branch of fat $k = 1$ black ringoids in $D \geq 8$ should not extend to vanishing area, but a horizon topology changing transition should be encountered to a respective branch of 'pinched' black holes.

While an analogous set of arguments should hold for the $k \geq 2$ black ringoids, the general picture Eugen Radu unveiled for these black objects represents only the tip of the iceberg, with innumerable further possibilities remaining to be studied. Still, he expressed his hope, that one may find a *periodic table of black objects*, i.e., a classification of all higherdimensional black objects based on a finite number of simple features.

Eugen Radu concluded his talk with a discussion of static solutions with horizon topology $S^2 \times S^{D-4}$ obtained previously, which possess a conical singularity^{40,41}. Allowing for further fields, in particular, a Maxwell field, he then addressed static

asymptotically flat magnetic configurations⁴², which can be viewed as generalizations of the $D = 5$ static dipole black ring⁴³. They can be balanced by “immersing” them in a background gauge field via a magnetic Harrison transformation^{42,44}. Since the magnetic field does not vanish asymptotically, the resulting configurations correspond to balanced black objects in a Melvin universe background⁴².

2.1.3. *Super-entropic black holes*

An intriguing new type of solutions was presented by David Kubiznak in his talk titled *Super-Entropic Black Holes*. Starting out with a prelude on the horizon topology, he first reminded the audience of Hawking’s theorem, requiring a spherical horizon topology for asymptotically flat black holes in four dimensions⁴⁵. He then turned to AdS black holes, where the horizons may be either compact or noncompact, and compact horizons may be Riemann surfaces of arbitrary genus g ⁴⁶, while noncompact horizons may, for instance, correspond to hyperboloid membranes⁴⁷, etc.

A very interesting new result here is the existence of black holes, which possess a noncompact horizon and at the same time a finite horizon area and therefore finite entropy. Such black holes were first discussed in^{24,25}, possessing horizons that are topologically $(D - 2)$ -spheres with two punctures. Thus there are even more surprising event horizon topologies possible.

Since these black holes^{24,25} may be viewed as a new type of ultraspinning limit of the Kerr-Newman-AdS solution, David Kubiznak turned to the general discussion of ultraspinning limits taken for Kerr-AdS black holes to obtain new types of black holes^{26,27}.

In the ultraspinning limit the rotation parameter a tends to the AdS radius l , yielding a singular limit, where three cases may be considered. i) In the brane limit (for $D > 5$), one keeps the physical mass fixed while simultaneously zooming in to the pole. This corresponds to the AdS analogue of the Gregory-Laflamme type instability of ultra-spinning Myers-Perry black holes^{14,48}. ii) In the hyperboloid membrane limit, the horizon radius is fixed, while simultaneously zooming in to the pole⁴⁷, yielding a rotating hyperboloid membrane with horizon topology $\mathbb{H} \times S^{D-4}$.

The so-called super-entropic limit iii) is obtained by first introducing a new azimuthal coordinate, by boosting the asymptotic rotation to the speed of light, and by compactifying the azimuthal direction. In four dimensions one then obtains a solution whose horizon is noncompact while being of finite size, which possesses an ergosphere, and no obvious pathologies, while the symmetry axis represents a boundary excised from the spacetime.

David Kubiznak then explained the name super-entropic. Since in AdS black hole spacetimes one can identify the negative cosmological constant with a positive pressure, one can also define the thermodynamic volume of a black hole^{49,50}. This led to the conjecture of the isoperimetric inequality, which can be interpreted as the statement that the entropy inside a horizon of a given volume is maximized for the

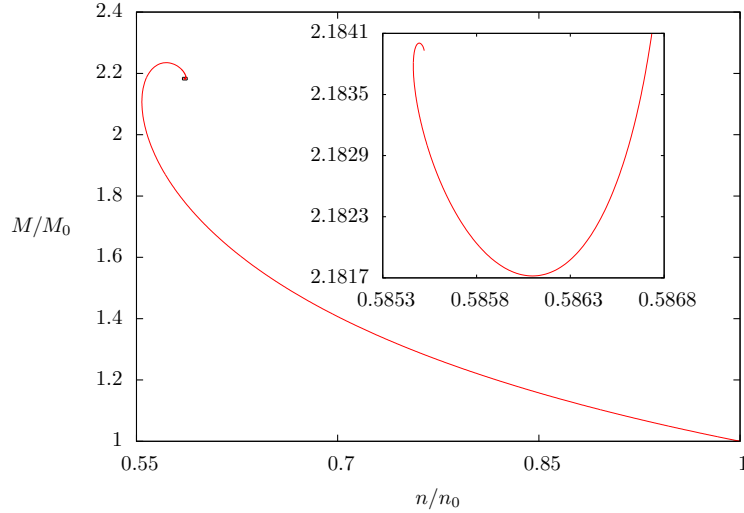


Fig. 1. Spiral of the scaled mass versus the scaled relative tension for nonuniform black strings in six dimensions (courtesy to Michael Kalisch).

Schwarzschild-AdS black hole⁵⁰. Since the new solutions provide a non-compact counterexample to this conjecture, they have been named super-entropic.

Subsequently, David Kubiznak applied the super-entropic limit analogously to the rotating Myers-Perry-AdS solutions in D dimensions^{51,52}, to obtain the corresponding higherdimensional type of solutions^{26,27}. Here the entropy per given thermodynamic volume exceeds the limit set by the conjectured isoperimetric inequality, at least in some parameter range, leading now to higherdimensional “super-entropic” black holes.

2.1.4. Black strings

Let us turn now to the case of compact extra dimensions and focus on the new results on nonuniform black strings⁶. These were presented by Michael Kalisch in his talk on *Highly Deformed Non-uniform Black Strings*.

In black strings the horizon completely wraps a compact extra dimension, with the horizon topology given by $S^{D-3} \times S^1$, where the S^1 now represents the compact dimension. While the branches of nonuniform black strings in five and six dimensions were constructed before²⁻⁵, the new calculations extend very much further towards the horizon topology changing merger^{7,8} with a branch of caged black holes⁹⁻¹².

The new results fully agree with the previous results in⁵, where a backbending of the branches had been noticed, when the mass, area or temperature were considered as functions of the relative tension. When considered as a function of the nonuniformity parameter $\lambda = \frac{1}{2}(R_{\max} - R_{\min})/R_{\min}$ this behavior is reflected in an oscillation of these physical quantities. While the previous calculations⁵ are reliable

up to $\lambda = 9$, the new calculations reach values of λ of several hundred.

As Michael Kalisch explained, the numerical scheme to obtain these highly accurate results is based on the use of a pseudo-spectral method. Very important ingredients are furthermore a set of appropriate coordinate transformations and the employment of many (up to seven) domains. For $\lambda \approx 200$, for instance, he demonstrated a most impressive accuracy of the calculations, yielding a value on the order of 10^{-13} for the residual and for the deviation from the Smarr formula, leaving the audience flabbergasted.

Thus with this method Michael Kalisch could not only confirm the occurrence of a maximum in the mass, but show two further extrema of the mass, and likewise for the area, the temperature, and the relative tension. The new results therefore represent the beginning of a spiral, when the mass, the area, and the temperature are considered versus the relative tension. These exciting results are demonstrated in Figure 1 for the scaled mass versus the scaled relative tension for nonuniform black strings in six dimensions (where the scaling is w.r.t. uniform black strings).

The calculations also strongly support the double-cone structure of the merger solution^{7,8}. To get a more complete understanding of the black string – caged black hole phase transitions, it would be highly desirable, to employ analogous sophisticated calculations also to the caged black hole branch, to see, whether such a spiral-like behavior arises there as well. Moreover, it would be very interesting to address with these superior numerical techniques also the horizon topology changing transition in more than six dimensions^{53–55}.

2.2. *Solutions with spherical horizon topology*

Let us now turn to the seemingly simple higherdimensional black holes with spherical horizon topology. Also here interesting new developments have taken place recently. In the session we have focussed on the effect which matter distributions and classical fields can have on the properties of such higherdimensional black holes.

2.2.1. *Black holes with distorted horizons*

Whereas the black holes discussed so far represent isolated black holes, one may also consider black holes which are not isolated but interacting with some external distribution of matter. While this would lead to dynamical systems in the general case, solvable only by numerical or perturbative methods, exact solutions of non-isolated black holes can be constructed as local solutions, physically relevant only in a close neighborhood of the black hole, which still incorporate information on the external matter fields.

Such distorted black holes were first obtained by Geroch and Hartle⁵⁶ for the static case in four dimensions, who also investigated various properties of these black holes. Rotating distorted black holes were constructed and investigated subsequently^{57–60}. As Petya Nedkova suggested in the introduction of her talk on

Rotating Distorted Black Holes in Higher Dimensions, such four-dimensional distorted black holes may, for instance, describe black holes surrounded by an accretion disk.

Petya Nedkova then continued to describe the general procedure on how to obtain distorted black holes, starting with a four-dimensional example. Expressing the metric in Weyl coordinates

$$ds^2 = -e^{2u} dt^2 + e^{-2u} \rho^2 d\phi^2 + e^{2(\gamma-u)} (d\rho^2 + dz^2) \quad (1)$$

with metric functions $u(\rho, z)$ and $\gamma(\rho, z)$, one finds the Laplace equation for u in three-dimensional flat space and a linear system for γ , which is always integrable for a given solution u . Since the uniqueness theorem does not extend to asymptotically non-flat vacuum solutions, one finds infinitely many black holes with a regular horizon, i.e., distorted black holes. The expansion coefficients in the solution for u then characterize the external matter distribution.

Turning to higherdimensional distorted black holes, Petya Nedkova recalled the known static solutions^{61,62}, and then addressed the construction of distorted Myers-Perry black holes with a single angular momentum⁶³. These solutions are obtained by applying a two-fold Bäcklund transformation on a five-dimensional vacuum Weyl solution as a seed, which describes a regular spacetime region in the presence of some external matter distribution.

This leads to five-dimensional stationary and axisymmetric vacuum solutions with a single rotation parameter. Their horizon has spherical topology, and the solutions are asymptotically non-flat. Therefore, Petya Nedkova interpreted the solutions as describing locally a Myers-Perry black hole in the presence of an external matter distribution. The solutions possess no curvature singularities outside their horizon, and conical singularities can be avoided by respecting a relation between the expansion coefficients of the metric functions. Petya Nedkova concluded her talk with a discussion of the physical properties of these distorted black hole solutions, considering in particular the ergoregion for dipole and quadrupole distortions.

2.2.2. *Einstein-Maxwell black holes*

Francisco Navarro-Lérida discussed in his talk the *Properties of Rotating Einstein-Maxwell-Dilaton Black Holes in Odd Dimensions*, restricting to asymptotic flatness and a spherical horizon topology. The general set of analytical Einstein-Maxwell-dilaton black holes can be constructed by employing a Kaluza-Klein reduction for a particular value of the dilaton coupling constant, $h = h_{KK}$ ^{64–67}. For different values of the dilaton coupling constant h , however, one either has to employ perturbative techniques^{68–74} or perform a numerical analysis^{75–78}, while for extremal black holes one may, in addition, resort to the near-horizon formalism to gain understanding of the properties of these black holes^{79–82}.

Specializing to black holes with equal angular momenta, Francisco Navarro-Lérida explained, that in odd- D dimensions this leads to an enhancement of the

symmetry of the solutions, and a substantial simplification of the equations to cohomogeneity-1, such that only ODEs and not PDEs must be considered⁷⁶, and that in the near-horizon formalism the resulting formulae are all analytical.

The results presented for the near-horizon formalism were quite intriguing. For the Einstein-Maxwell case ($h = 0$), there are two sets of near-horizon solutions for all odd dimensions. Denoting them as the Myers-Perry branch and the Reissner-Nordström branch, since they emerge from the extremal Myers-Perry and Reissner-Nordström solutions, respectively, Francisco Navarro-Lérida pointed out, that for the black holes on the Myers-Perry branch the angular momenta are proportional to the horizon area, while for the black holes on the Reissner-Nordström branch the angular momenta are proportional to the horizon angular momenta. The two branches of near-horizon solution cross at a critical point.

Interestingly, the numerically obtained global solutions do not exist for the full near-horizon branches. Instead, only particular parts of these near-horizon branches are realized globally. Thus the family of global solutions consists only of the first part of the Myers-Perry branch and the second part of the Reissner-Nordström branch, meeting at the critical point.

In contrast, for an arbitrary finite value of the dilaton coupling constant h , there is only a single set of near-horizon solutions, and the angular momenta are always proportional to the horizon area, as Francisco Navarro-Lérida explained. He then addressed the domain of existence of Einstein-Maxwell-dilaton black holes, showing that it is determined by the set of static black holes and the set of extremal rotating black holes. Finally he noted, that one can infer to good approximation the horizon angular velocity of the extremal rotating black holes from the surface gravity of the static black holes.

2.2.3. *Einstein-Maxwell-Chern-Simons black holes*

Even more surprising results for extremal charged rotating black holes in higher dimensions were reported by Jose Blázquez-Salcedo in his talk *Charged and Rotating Black Holes in 5D Einstein-Maxwell-Chern-Simons Theory*. In odd spacetime dimensions one may add a Chern-Simons term to the Einstein-Maxwell action. In the presence of this term analytical black hole solutions were found in five dimensions, when the Chern-Simons coupling constant assumes a particular value $\lambda = \lambda_{\text{sg}}$ corresponding to minimal supergravity^{83–85}.

For values of the Chern-Simons coupling constant $\lambda > \lambda_{\text{sg}}$, however, intriguing new behavior occurs^{28,86–88}. For instance, counterrotation sets in, where the horizon angular velocity and the angular momentum have opposite signs^{86,87}, giving rise to an instability^{86,89}. For $\lambda > 2\lambda_{\text{sg}}$ uniqueness of the black hole solutions is violated and black holes with a rotating horizon but with vanishing angular momenta, $J = 0$, arise⁸⁶.

To this set of remarkable properties of Einstein-Chern-Simons black holes, Jose Blázquez-Salcedo added surprising new results in his talk. He first analyzed the

extremal black hole solutions in the near-horizon formalism for $\lambda > 2\lambda_{\text{sg}}$, showing that for positive charge there are three distinct branches, while there is a single branch for negative charge. In an area versus angular momentum diagram, he then pointed out several particular solutions including the extremal Reissner-Nordström solution, a set of two solutions with $J = 0$, a set of two singular solutions with vanishing area, and two more cusp solutions. Subsequently, he compared with the numerically obtained global solutions, and showed that they yield a far more intricate phase diagram than the near-horizon solutions. Consequently, a given near horizon solution can correspond to either a single global solution, to more than one global solution, or to no global solution at all^{28,88}.

Moreover, as Jose Blázquez-Salcedo explained, there is a whole sequence of extremal rotating global $J = 0$ solutions, and not only two as suggested by the near-horizon formalism. These solutions possess an increasing number of radial nodes in a metric function and a gauge field function, as he demonstrated for solutions with more than 30 nodes^{28,88}. Fixing the charge, the mass of this sequence converges to the mass of the corresponding extremal Reissner-Nordström black hole. He showed, that these excited extremal black hole solutions are located inside the domain of existence. Therefore these Einstein-Maxwell-Chern-Simons black holes exhibit a new type of uniqueness violation, namely for the same sets of global charges there can exist both extremal and non-extremal black holes^{28,88}. Jose Blázquez-Salcedo ended with a glance at his current work, which considers the presence of a negative cosmological constant.

2.2.4. *p*-form black holes

Marcello Ortaggio addressed higherdimensional black holes in the presence of a p -form field instead of the standard electromagnetic 2-form field, while including also a cosmological constant. He started by explaining the class of four-dimensional exact solutions given by the Robinson-Trautman family⁹³, which is defined by the existence of a geodesic and shear-free, twist-free, expanding null vector field. This family includes static black holes with a cosmological constant, the C -metric, radiation, the Vaidya metric, etc. He then recalled electrovac Robinson-Trautman spacetimes which describe the formation of black holes by gravitational collapse of electromagnetic radiation^{94–97}.

Turning to Robinson-Trautmann spacetimes in higher dimensions, Marcello Ortaggio recalled that the general metric is known⁹⁸, and that the electrovac solutions essentially reduce to static black holes with an Einstein horizon^{98,99}. He remarked that the shearfree condition may be too strong in higher dimensions^{100–103}, and pointed out that in the case of more general p -forms electromagnetic radiation may have different properties than in the standard case^{104,105}.

Focussing on the Einstein-Maxwell equations with p -forms, Marcello Ortaggio recalled a number of known results for static p -forms like the no dipole hair theorem¹⁰⁶. Then he described the construction of *Static and Radiating p-form Black*

*Holes in the Higher Dimensional Robinson-Trautman Class*¹⁰⁷. He determined the Robinson-Trautman spacetimes in the presence of a Maxwell p -form, but he restricted to the case of aligned Maxwell fields¹⁰⁷.

Marcello Ortaggio explained that the properties of these solutions depend strongly on the dimension D and the value of p . In odd dimensions one finds static black holes with an electromagnetic field¹⁰⁸, while in even dimensions static black holes are also present, but there are in addition non-static metrics¹⁰⁷. For $D = 2p$, some solutions may describe black hole formation by the collapse of electromagnetic radiation¹⁰⁷ analogously to⁹⁴.

2.3. Properties of solutions

Some of the talks did not present new black hole solutions but were solely devoted to the discussion of the physical and mathematical properties of known higherdimensional black holes, including their geodesics, stability and greybody factors.

2.3.1. Geodesics

For our understanding of the physical properties of black holes it is essential to study the motion of test particles and light in these spacetimes. As shown in^{109–111}, the geodesic equations of the higherdimensional Myers-Perry black holes are separable. The geodesics in Myers-Perry black hole space-times were studied in^{112–116}. In contrast, the equations of motion in black ring spacetimes were found to be separable only in special cases^{117–125}.

In the presence of a cosmological constant, the geodesics of static black holes in four and higher dimensions^{114,126} and of rotating Kerr-AdS black holes¹²⁷ are known exactly for the general case. In his talk *Geodesics of the AdS Myers Perry Black Hole with Equal Angular Momenta* based on¹²⁸ Terence Delsate analyzed the timelike and null geodesics of Myers-Perry black holes with AdS asymptotics^{51,52}. To facilitate the analysis, he restricted to black holes with equal angular momenta, and he focussed on spherical orbits.

In the discussion of his results, Terence Delsate then addressed the timelike innermost stable circular orbits and the null circular orbits. Interestingly, unlike their asymptotically flat counterparts, these higherdimensional black holes allow for bound timelike orbits. Terence Delsate then explained that in these AdS spacetimes, for sufficiently massive black holes there is a parameter range where the spacetime is stable against superradiance and the innermost stable circular orbit is located inside the ergoregion. In contrast, for the massless case, he could not find stable circular orbits outside the horizon. However, there could be stable null circular orbits around a naked singularity¹²⁸.

2.3.2. Greybody factors

Hawking radiation emitted at the event horizon of a black hole gets modified by the black hole geometry, yielding a spectrum for an asymptotic observer which is no longer a black body spectrum, with the difference being encoded in the greybody factor (see e.g. ref.¹²⁹ and references therein). In his talk *Greybody Factors of Rotating Cohomogeneity-1 Black Holes* Ednilton de Oliveira analyzed the greybody factors of black holes in higher odd dimensions, rotating with equal angular momenta, for different asymptotics: asymptotically flat black holes, de Sitter and anti-de Sitter black holes¹³⁰. His objectives were to understand how the cosmological constant influences the greybody factors of rotating black holes and to analyze superradiance.

Ednilton de Oliveira pointed to the difficulty of this problem, revealing itself in the scarcity of results on greybody factors for rotating and non-asymptotically flat black holes. For static higherdimensional black holes a proper definition of greybody factors for both asymptotically de Sitter and anti-de Sitter spacetimes was provided in¹³¹, where the greybody factors were obtained for scalar fields by the use of approximate analytic methods, valid in certain regimes of the parameters (such as s -waves and low frequencies). Since the methods of¹³¹ can also be applied to more general black hole spacetimes, which depend on a single (radial) coordinate, this suggested the study of the greybody factors of rotating black holes in odd dimensions with equal angular momenta.

After recalling these cohomogeneity-1 black hole spacetimes, Ednilton de Oliveira considered a minimally coupled massless scalar field. Showing that a separation ansatz leads to a simple angular eigenvalue equation, he concentrated on the radial equation and the effective potential for the three different asymptotic cases. He recalled the approximate analytic method¹³¹ and its range of validity, addressed the numerical method employed in his calculations, and then presented the main results¹³⁰. The analytic results for the s -wave modes include the observations that for asymptotically flat black holes and low frequencies ω the greybody factors behave like ω^{D-2} . For small de Sitter black holes the greybody factor is given by the ratio of the areas of the black hole and the de Sitter horizon for small frequencies, while for small anti-de Sitter black holes the greybody spectrum exhibits a rich structure with large amplitude oscillations¹³⁰. The numerical calculations showed, that the analytic approximations become better with increasing dimension. They also revealed that the strongest superradiance effect is seen for p -waves, but decreases with increasing dimension¹³⁰.

3. Solutions in Lovelock gravity

Lovelock gravity represents a gravity theory generalizing Einstein's General Relativity to higher dimensions¹³². It is based on a symmetric metric tensor endowed with a Levi-Civita connection. The operator on the r.h.s. of the field equations is divergence free, and the field equations are of second order. While in four dimen-

sions these requirements lead to General Relativity with a cosmological constant, in higher dimensions further terms are allowed as shown by Lovelock¹³². In five and higher dimensions, for instance, the Gauss-Bonnet term can be added, yielding Einstein-Gauss-Bonnet theory. Reviews on black holes in Lovelock gravity can, e.g., be found in refs.^{133–135}. Note, that recently arguments were put forward¹³⁶ showing that EGB theory is in conflict with causality, unless it has a UV completion involving an infinite tower of higher-spin particles with fine-tuned coupling.

3.1. *Black holes in Lovelock gravity*

While general Lovelock theories include all terms allowed for a given dimension in the action, thus combining the action of General Relativity with higher order Lovelock terms, in pure Lovelock theory the Einstein-Hilbert action is present only in three and four dimensions, whereas in higher dimensions always only the respective Lovelock term \mathcal{L}_N , characteristic for the dimension D , where $N = \lfloor (D-1)/2 \rfloor$, together with the cosmological constant is considered^{137–140}.

3.1.1. *Pure Lovelock black holes*

In his talk on *Gravity in Higher Dimensions*^{139,140} Naresh Dadhich first explained how General Relativity would follow from the geometric properties of Riemann curvature, recalling that the Einstein tensor is non-trivial only in $D > 2$ dimensions, while gravity is purely kinematic in $D = 3$, and a non-trivial vacuum solution for free propagation requires $D > 3$.

Naresh Dadhich then argued, that in order to universalize the kinematic property of General Relativity to all odd dimensions, requiring a generalization of General Relativity for higher dimensions, pure Lovelock gravity is uniquely singled out. In his view pure Lovelock gravity is the most natural generalization of General Relativity because it uniquely retains the second order field equations, while its action is a homogeneous polynomial built from the Riemann tensor^{137–140}. Indeed, in all odd dimensions $D = 2N + 1$, in pure Lovelock theory gravity is kinematic^{141,142}. He then argued that the pure Lovelock theory based on the N -th Lovelock term is only valid in $D = 2N + 1$ odd dimensions and $D = 2N + 2$ even dimensions, including General Relativity for $N = 1$.

Turning to black holes, Naresh Dadhich reminded the audience of the BTZ black hole in three dimensions¹⁴³, arguing that analogs of BTZ black holes should exist in pure Lovelock gravity in all odd dimensions¹⁴¹. Indeed, inspection of the static spherically symmetric black holes of pure Lovelock theory^{137,145} reveals their presence. Inspection of the general set of static black hole solutions in pure Lovelock gravity shows, that the gravitational potential in the metric function is essentially the N -th root of the potential in General Relativity, both in odd and even dimensions^{137,144,145}. Asymptotically, this yields a $1/r^{D-3}$ dependence of the potential as for the black holes of General Relativity. In even dimensions for small values of

the radial coordinate the potential is of order $1/r^{1/N}$, which allows for stable bound orbits in pure Lovelock black hole spacetimes in even dimensions. Moreover, the characteristic gravitational potential leads to a remarkable universal thermodynamical behavior in terms of the event horizon radius. In particular, the temperature and the entropy always bear the same relation to the horizon radius in all odd and even dimensions^{137,144,145}.

3.1.2. *Black holes with nonspherical horizon topology*

In his talk entitled *Static Pure Lovelock Black Hole Solutions with Horizon Topology $S(n) \times S(n)$* , Josep Pons recalled that in order to obtain spherically symmetric static black holes in Lovelock gravity, one basically has to solve an algebraic polynomial of degree N ^{144,148–153}. He then argued that the algebraic character of the ultimate equation holds also for $S(n) \times S(n)$ black holes, where the horizon topology is a product of two spheres^{146,147}.

Starting out with such solutions in General Relativity Josep Pons recalled the solutions of type $S(d_0) \times S(d_0)$, $D = 2d_0 + 2$, reminding the audience of the $S^2 \times S^2$ merger solution of Kol in the context of the nonuniform black string – caged black hole transition⁷.

In Einstein-Gauss-Bonnet gravity an interesting example of a black hole solution with an $S(d_0) \times S(d_0)$ topology was also studied before^{146,154–157}. Besides having a nontrivial boundary, the spacetime contains a contribution from the Weyl tensor, which leads to a slow falloff of the metric function. Josep Pons explained that the presence of this term implies that the cosmological constant should always be present and positive. Moreover, in addition to the central singularity there also occurs a noncentral singularity, which may be naked. The latter can be avoided when the black hole mass and the cosmological constant satisfy a certain constraint^{146,147}.

Restricting to the pure Lovelock case, Josep Pons then considered black holes with horizon topology $S(N) \times S(N)$ with $D = 2N + 2$. In fact, a general pattern appeared in this case, namely for even N , the cosmological constant should be positive, yielding a black hole horizon and a cosmological horizon (within a certain range of parameters), whereas for odd N the cosmological constant should be negative to obtain a black hole horizon^{146,147}. Interestingly, $S(N) \times S(N)$ black holes are thermodynamically stable for odd N and negative cosmological constant, and unstable for even N and positive cosmological constant. Moreover, the universal thermodynamical behavior of pure Lovelock black holes with spherical horizon topology^{137,144,145} continues to hold also for these $S(N) \times S(N)$ black holes^{146,147}.

3.2. *Properties of Lovelock black holes*

In the last part of the session, the properties of known Lovelock solutions were discussed, in particular, their thermodynamics and stability.

3.2.1. *Stability of black strings*

Julio Oliva addressed black strings in Lovelock gravity in his talk entitled *Black Strings in Gauss-Bonnet Theory are Unstable* based on¹⁵⁸. He first recalled the analysis of the Gregory-Laflamme instability of black strings in General Relativity in five dimensions¹, as well as the Gubser-Mitra conjecture, relating the thermal and perturbative instabilities of black holes with extended directions^{159,160}.

Subsequently Julio Oliva turned to black strings in Einstein-Gauss-Bonnet theory, and analyzed the behavior of the metric function of the analytically known black hole solutions in D dimensions¹⁴⁸. In particular, he discussed the two limiting cases, where, depending on the ratio of the radial coordinate and the square root of the Gauss-Bonnet coupling constant, either a Schwarzschild-Tangherlini solution is approached or a solution of the pure Gauss-Bonnet theory (without cosmological constant)¹⁵³.

Pointing out that the latter solutions can be oxidated to construct homogeneous black string and p -brane solutions¹⁶¹, Julio Oliva then addressed pure Gauss-Bonnet black strings (without cosmological constant). He recalled their thermal instability¹⁶¹, and argued, that this instability should have a perturbative counterpart. Proceeding analogously to the Gregory-Laflamme instability analysis in General Relativity, he then analyzed the perturbative instability of pure Gauss-Bonnet black strings in seven dimensions, and came to the analogous picture concerning the instability of the black string in pure Gauss-Bonnet theory as the one known from General Relativity¹⁵⁸.

3.2.2. *Thermodynamics*

In her talk *Lovelock Black Hole Thermodynamics* Antonia Frassino addressed phase transitions of static spherically symmetric and asymptotically AdS black holes in Maxwell-Lovelock gravity, focussing on 2nd and 3rd order Lovelock terms¹⁶² (for earlier work see e.g.^{163–167}). She first recalled that by identifying the negative cosmological constant with a thermodynamical pressure, and introducing an associated conjugate thermodynamic volume one obtains a generalized first law and Smarr formula both in General Relativity⁴⁹ and in Lovelock gravity^{168,169}.

In her thermodynamical investigations Antonia Frassino then kept the Lovelock coupling constants fixed except for the cosmological constant and studied the possible phase transitions based on the behavior of the Gibbs free energy in the canonical ensemble. She pointed out that the pressure must be sufficiently small in these solutions, since otherwise the solutions would not possess an asymptotic AdS region, and space would become compact.

While previous studies^{170–175} of the Gauss-Bonnet (2nd order Lovelock) case had already shown a van der Waals behavior¹⁷⁶, as well as reentrant phase transitions and tricriticality, Antonia Frassino added that a triple point arises only in six dimensions and has no counterpart in higher dimensions¹⁶². She then explained

her new results for the 3rd order Lovelock case, where for hyperbolic Lovelock black holes multiple reentrant phase transitions occur, and for special tuned Lovelock couplings a new type of isolated critical point appears¹⁶².

4. Conclusions

The field of higherdimensional black holes has seen most interesting developments during the last years. Besides much progress with analytical methods also major progress concerning numerical methods has been achieved. Here in particular the highly sophisticated pseudospectral methods promise to yield further impressive results in the near future, which will hopefully be reported in this session at MG15.

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